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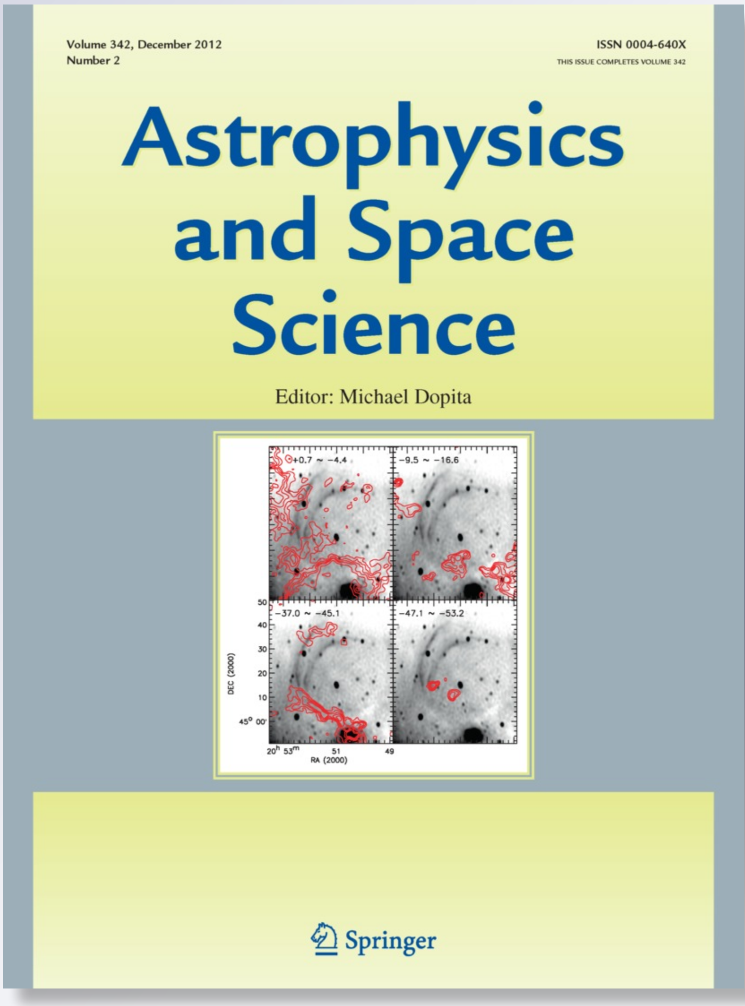

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ORIGINAL ARTICLE

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Abstract The contemporary notion of black hole originates in Oppenheimer and Snyder's 1939 article "On Continued Gravitational Contraction" (Phys. Rev. 56:455, 1939). In particular, Penrose (Phys. Rev. Lett. 14:57, 1965) showed that their metric gave rise to trapped surfaces, that is regions of space from which no light rays can escape, and proved that within such surfaces black-hole formation is inevitable. Section "No trapped surfaces" of this article shows that a simple modification of the Oppenheimer-Snyder metric, fully consistent with General Relativity, may be made, so that all radial light rays originating in the interior escape to the exterior. There is no trapped surface and no black hole; on the contrary there is a stable end state with finite density, contained within a sphere of Schwarzschild radius. Implications for the interpretation of General Relativity, and also for experimental observation of supermassive objects and the Event Horizon Telescope project, are discussed in the concluding section.

Keywords Gravitational collapse · Causality · Gravitational field · Trapped surface · Oppenheimer-Snyder · Event Horizon Telescope

1 Introduction

Oppenheimer and Snyder (1939) (OS) constructed a sphero-symmetric metric to represent the gravitational collapse of an idealized material, or "dust", in which there are no forces other than gravity, and therefore zero pressure. They did not attempt to trace the orbits of individual dust particles, but in choosing their article's title, "On continued gravitational contraction", and also in stating that their metric confirmed the conclusion of Oppenheimer and Volkoff (1939) (OV), namely that no object heavier than about two Suns could avoid total collapse, they opened the way for their successors to claim that OS had established the inevitability of black holes.

What they did show was that, in their metric, there are regions of space from which no light rays can escape, which led Penrose (1965) to the notion of a trapped surface and to deduce that inside such a surface a singularity forms. Shortly after that such singularities came to be called black holes (Thorne 1994). The present article establishes that only a small modification of OS is required in order to remove the trapped surface. Also this modified metric, fully consistent with General Relativity (GR), gives a stable state as the time coordinate goes to plus infinity. These results indicate the existence of stable supermassive objects of finite density; indeed the more massive they are, the lower is their density.

Note that there have been other recent criticisms of OS, from which some of the ideas in the present article originate, namely the insistence on Hilbert causality (Logunov and Mestvirishvili 2012) and the recognition of trapped surfaces as the most radical departure from causality (Mitra 2011). My study shows, however, that the OS approach is sound when their metric is suitably modified, and runs counter to the earlier OV one. It is the latter, therefore, which is the more open to criticism on the grounds of causality.

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2 No trapped surfaces

In dimensionless form (r in units of $2m$, R in units of R_0) the OS metric may be expressed as

$$ds^2 = (\sqrt{R}dR - \sqrt{r}dr)^2 - (R/r)dR^2 - r^2\big(d\theta^2 + \sin^2\theta d\phi^2\big) \quad (1)$$

for $R > 1$, and

$$ds^2 = \big(r^3/R^3\big)(dR/R - dr/r)^2 - \big(r^2/R^2\big)dR^2 - r^2\big(d\theta^2 + \sin^2\theta d\phi^2\big) \quad (2)$$

for $R < 1$. Here the coordinate R is comoving, that is for $R > 1$ a freefalling test particle and for $R < 1$ a dust particle moves along a geodesic whose equation is $R =$ constant. In the exterior region, $R > 1$, the OS metric may be transformed into that of Schwarzschild, that is

$$ds^2 = \frac{r-1}{r}dt^2 - \frac{r}{r-1}dr^2 - r^2\big(d\theta^2 + \sin^2\theta d\phi^2\big) \quad (R > 1), \quad (3)$$

by putting

$$t = \frac{2}{3}R^{3/2} - \frac{2}{3}r^{3/2} - 2\sqrt{r} + \ln\frac{\sqrt{r}+1}{\sqrt{r}-1} \quad (R > 1). \quad (4)$$

OS obtained a match from the interior to the exterior metric with

$$t = \frac{2}{3} - \frac{2}{3}y^{3/2} - 2\sqrt{y} + \ln\frac{\sqrt{y}+1}{\sqrt{y}-1} \quad (R < 1), \quad (5)$$

and $y(R, r)$ satisfying $y(1, r) = r$, and with $\partial y/\partial R$ at $R = 1$ chosen so that the metric is continuous[1] there, that is

$$g_{tt} = \frac{r-1}{r}, \qquad g_{tr} = 0, \qquad g_{rr} = -\frac{r}{r-1} \quad (R = 1). \quad (6)$$

Note that all θ and ϕ components of $g_{\mu\nu}$ remain unchanged and continuous under the mapping from R to t. However, without offering any justification or explanation, OS strengthened the latter condition by requiring, in addition, that

$$g_{tr} = 0 \quad (R < 1), \quad (7)$$

and arrived "uniquely" at the solution

$$y = \frac{r}{R} + \frac{R^2}{2} - \frac{1}{2}. \quad (8)$$

The interior metric for this choice of y is obtained by making the substitution

$$dR = \frac{R}{r - R^3}\left(\frac{R(y-1)}{y^{3/2}}dt - dr\right) \quad (9)$$

in (2), giving

$$ds^2 = \frac{(y-1)^2r^2}{Ry^3(r-R^3)}dt^2 - \frac{r}{(r-R^3)}dr^2 - r^2\big(d\theta^2 + \sin^2\theta d\phi^2\big). \quad (10)$$

I shall show that this choice of y is responsible for Penrose's trapped surface. On the other hand, the simple modification

$$y = \frac{r}{R} - \frac{5}{4} + \frac{3R}{2} - \frac{R^2}{4} \quad (11)$$

gives the interior metric

$$ds^2 = \frac{4(y-1)^2}{(2r + R^3 - 3R^2)^2} \times \left[\frac{r^2(r-R^3)}{Ry^3}dt^2 - \frac{rR}{y-1}dr^2 + \frac{r^2(1-R)}{y^{3/2}(y-1)}drdt\right] - r^2\big(d\theta^2 + \sin^2\theta d\phi^2\big) \quad (12)$$

and this satisfies (6) but *has no trapped surfaces.*

To establish the latter result, consider a radial light ray through the event $(R, r) = (1, y_1)$ with $y_1 > 1$. In the interior its equation is the null geodesic

$$\frac{dR}{R} - \frac{dr}{r} = \sqrt{\frac{R}{r}}dR \quad (13)$$

with solution

$$r = R\left(\sqrt{y_1} + \frac{1-R}{2}\right)^2. \quad (14)$$

By substituting (11) and (5), this gives r as a (monotonic) function of t. Because the interior and exterior metrics satisfy the matching condition (6), it joins smoothly to an exterior null geodesic satisfying

$$dr = \frac{r-1}{r}dt, \quad (15)$$

which integrates to give

[1] In terms of (R, r) the tensor g appears to be discontinuous at $R = 1$, but this discontinuity is removed when we take account of the different definitions of $t(R, r)$ in the two regions. We identified incorrectly this discontinuity as the "fatal error" of OS in earlier articles (Marshall 2007, 2009; Marshall and Wallis 2010). The real fatal error of OS was their imposition of (7).

$$t = r - y_1 + \ln \frac{r-1}{y_1 - 1} + \frac{2}{3} - \frac{2}{3} y_1^{3/2} - 2\sqrt{y_1} + \ln \frac{\sqrt{y_1}+1}{\sqrt{y_1}-1}. \tag{16}$$

Thus every event (t, r) with $r > 0$, interior and exterior, lies on a unique null geodesic specified by y_1, and may be reached by a light ray starting at $r = 0$, after a finite time interval.

We have therefore established that the OS metric, with the modification (11), collapses to a final state without singularity and without trapped surfaces. The final state ($t \to +\infty$) corresponds to $y = 1$, that is

$$r_{\lim}(R) = \frac{R}{4}(3 - R)^2 \quad (R < 1). \tag{17}$$

This does not show a concentration of stellar material at $r = 0$, as hinted at in the title of the OS article and stated explicitly by almost all subsequent commentators. Rather it shows a concentration near $r = 1$, as may be demonstrated by considering that a ball of radius $R = 2^{-1/3} = .7937\ldots$, containing half of the total material in the early stages of collapse,[2] maps into one of radius $r_{\lim} = .9658\ldots$. Since $R = 1$ goes to $r_{\lim} = 1$, this means that half of the stellar material ends up in a thin surface shell.

In Fig. 1 I have traced some of the particle and null geodesics for the amended OS metric. One feature to note is that, though all light rays go from the interior to the exterior in finite time, the speed of light tends to zero as the value of t tends to plus infinity, just like the speed of a free falling test particle, as it approaches the surface $r = 1$. Both of these are consequences of the infinite red shift at the surface, noted already by OS. The results obtained here confirm those of Penrose; there is a connexion between the presence or absence of a trapped surface and the presence or absence of a singularity. The trapped surface owes its origin to the fact that the last light ray to escape to the exterior is the null geodesic through $(R, r) = (1, 1)$, which leaves $r = 0$ with $r/R = 9/4$. That corresponds to $y = 7/4$ if we use the OS match (8) but $y = 1$ if we use the modified match (11). The latter case corresponds to $t = +\infty$, meaning that all rays reach the exterior, while the former corresponds, in our units, to $t = -1.5491$, meaning that all rays emitted from $r = 0$ after this time are trapped. So, in the latter case there is no singularity, and in the former case there is a singularity.

A referee has suggested that the imposition of condition (7) by OS may have been dictated by some requirement on the material stress tensor $T^{\mu\nu}$ and the comoving nature of the coordinate R. These were key features in their construction of the metric (2), which they expressed in the coordinates (τ, R, θ, ϕ), τ being the particle's proper time; in these

[2]OS showed, from analysis of the mass tensor, that for $t \to -\infty$ the mass density is uniform in $R < R_0$.

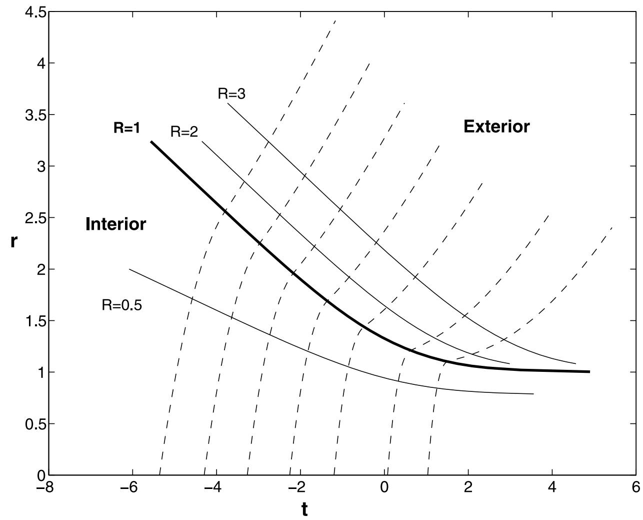

Fig. 1 The map of space-time for the amended OS metric, with particle geodesics depicted by continuous and light geodesics by broken lines. The geodesic $R = 1$ is the boundary between exterior and interior regions. All points of the space are causally connected at all times, and in particular any radial light ray originating in the interior region eventually escapes into the exterior region

coordinates both the metric tensor $g^{\mu\nu}$ and $T^{\mu\nu}$ are diagonal, but neither is diagonal in (r, R, θ, ϕ). To investigate whether imposing diagonality on $g^{\mu\nu}$ in (t, r, θ, ϕ) implies any special property in $T^{\mu\nu}$, I constructed the latter tensor using both the OS and my expressions for t in terms of r and R; in neither case is $T^{\mu\nu}$ diagonal. Furthermore, as noted in my previous article (Marshall 2007), the OS function $t(r, R)$ in (8) gives a dependence of r on R as t tends to plus infinity, very similar to what we reported above using (11); this suggests that, even with the OS choice of metric in the (t, r) coordinates, a fuller investigation of the tensor $T^{\mu\nu}$ will reveal a stable end state, thereby contradicting the earlier analysis of Oppenheimer and Volkoff (1939).

3 Conclusion

Apart from the above brief examination of the stress tensor, my analysis up to this point was purely differential geometry; even within a strictly geometric interpretation of GR, there are grounds for challenging the universality of Penrose's theorem (Penrose 1965).

But what are the implications for experimental astronomy? The supermassive object at the centre of our galaxy offers a prime test of the gravitationally collapsed structure proposed in this article. The event horizon telescope project, to probe light rays passing close to the Schwarzschild radius, would also detect any rays which penetrate the surface shell.

An objective reader may, perhaps, be tempted to say that the matching choices (8) and (11) are of equal validity, and that we may decide between them only on the basis of such experimental observation. Or again he may declare a liking for one or the other of these on the basis of intuition. The latter is notoriously subjective, but nevertheless, when the intuitions of a learned community coincide the epithet "counterintuitive" can be pretty powerful. Either of the proposed matching choices may be classified as counterintuitive, depending upon our point of view.

Since about 1970 it has become customary, following a lead of Hawking (Hawking and Ellis 1973) , to refer to the set of events lying on trapped rays as the interior of an *absolute horizon,* and the whole set of such concepts – trapped surfaces and horizons – gives rise to the abandonment of causality and its replacement by teleology (effect preceding cause, see Thorne 1994, pp. 414–418). So, if we are strongly attached to causality as the basis of scientific analysis, we should reject (8) on the ground that it is counterintuitive.

On the other hand there is the analysis, made by many generations going back to Oppenheimer and Volkoff (1939) that, above a certain density, "no force can countervail against gravity". Clearly OS must have been a prisoner of that intuition, otherwise they would surely have discovered the alternative (11). But the latter leads us to conclude that there is a strong concentration of dust near the surface, so what force pushes it there? Since there is no force other than gravity, it must be gravity that pushes it there; *at sufficiently high density gravity becomes repulsive.* So should we reject (11) because it is counterintuitive?

We have argued elsewhere (Marshall 2007, 2009, 2011, Marshall and Wallis 2010) that a rather large change of intuition is required. We need to return to the idea that gravity, like electromagnetism, is a field, that is it is more than just a modification of flat-space geometry. As a field, it has an associated energy-momentum tensor (not a pseudotensor); its energy density may be negative, and since energy is mass, and all mass gravitates, *a high concentration of gravitational energy leads to gravitational repulsion.* As long as we insist that GR is a purely geometric theory, underpinned by a strong Equivalence Principle which admits of no privileged coordinate system, such ideas are ruled out. There is, however, a long established field interpretation of GR, with its origins in Einstein's article on gravitational waves (Einstein 1918) and developed in the text of Weinberg (1972), especially the Preface and "the geometrical analogy" on page 147. There are some more recent developments of the field interpretation which argue for a privileged coordinate system; these may be presented as being either within (Babak and Grishchuk 1999) or outside GR (Logunov 2001).

If we accept that a field interpretation of GR is valid, then this implies (Marshall 2011) that the metric choice (11) is only one of a substantially larger family. The change from (8) to (11) is fully consistent with GR, but we may have to go outside GR to decide which of them is the less counterintuitive!

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